

with the name of Canard. Under the head of incidence the case of a strict monopoly is said to be of sufficient importance to receive some special notice. Professor Bastable justly observes that as "in very few cases is a monopoly strictly so called to be found," "the theoretical conception of a pure monopoly is of little direct service in dealing with the question of incidence." Accordingly, he is not to be understood as attaching much importance to one or two specimens which he gives of abstract reasoning about this hypothetical case: *e.g.*, "if a slight rise of price seriously checks consumption, or in other words, if the demand is elastic the monopolist suffers more than in the case of inelastic demand."

Among the recent developments of fiscal policy noticed in the third edition are the new systems of succession duties introduced into France in 1901 and extended in 1902, according to which the duty on inheritance above £2,000,000 payable by relatives of more remote degrees and strangers amounts to 20·5 per cent. The scale of duties is pronounced by Professor Bastable "instructive as showing the arbitrary way in which progressive taxation can be applied." He also notices alterations of fiscal policy in Austria, in the United States, and in Spain. He observes with misgiving the changes which have supervened in our own country, the grave problem for the future constituted by the growth of expenditure, the indications of "a disposition on the part of the predominant political party to depart from the financial principles which have prevailed since 1860." Professor Bastable not only indicates a danger, he has also given us the best means of averting it—sound principles of Public Finance.

Encyklopädie der mathematischen Wissenschaften. Band I, Heft 6. *Anwendungen der Wahrscheinlichkeitsrechnung auf Statistik.* Von LADISLAUS V. BORTKIEWIEZ. Band I. Heft 7. *Anwendungen der Mathematik auf Nationalökonomie.* Von V. PARETO. 1903.

The German Encyclopædia of Mathematical Sciences has very properly included among its constituent treatises two relating to those branches of the human or moral sciences which require the use of mathematics, namely, the more abstract portions of political economy and statistics.

The scope and function of the mathematical method in political economy are indicated by Professor Pareto with that persuasive lucidity which characterises his introduction to the subject in his articles in the *Giornale degli Economisti* and subsequent writings,

among which we take the opportunity of calling attention to the résumé of the course of lectures given at the École des Hautes Études Sociales in Paris (1901-2). The fundamental equations of exchange are derived by Professor Pareto from the conception of utility, or, as he prefers to say, *ophelimity*, which in the mechanics of trade plays nearly the same rôle as the concept of force in mathematical physics. "With the equations given for exchange we must combine the equations of production and capitalisation, and thus we obtain the system of equations which completely determine the economic cycle for the case of free competition." The *simultaneity* of these equations was first clearly conceived by Professor Walras, the distinguished predecessor of Professor Pareto in the Chair of Political Economy at Lausanne. Without this conception, as Professor Pareto says, "useful detailed studies" may be attained, "but no insight into the whole system." When the prices, say p_1 , p_2 , etc., considered as unknown quantities, depend upon constants entering into several simultaneous equations, it is impossible to specify the particular constant which "determines," p_1 for example, and "it is an unfruitful controversy when one party maintains that this constant is a_2 another party that it is a_1 ." The unmathematical economist vainly seeks some single attribute as the condition of price or ground of interest.

Among Professor Pareto's original contributions to the subject we may notice his study on the quantitative data with which the mathematical economist has to deal. As we understand Professor Pareto, these data do not comprise measurements of utility: psychical quantities, unlike physical, cannot be expressed as the sum of so many units. The exercise of *choice*, the preference of the economic man for one combination of goods to another, results in a system of *indifference-curves* which are comparable with the isobars or isotherms of physical science in that each successive curve denotes a greater intensity of the attribute under consideration, but differ in that the economic, unlike the physical, curves cannot be each labelled with a number. There is also to be noticed our author's claim to have proved by mathematical reasoning "what is by no means evident *a priori*," that the methods of production, the values of the "production coefficients" would be the same in an intelligent regime of socialism, as it is in the system of *laissez-faire*. The dynamics of the economic system offer an attractive subject to one who is penetrated with the thought that "the analogies between mathematical economics and pure mechanics are numerous and far-reaching (*tiefgründend*)." Elsewhere Professor Pareto has made reconnaissances in the almost

unexplored region of economic *dynamics*, understanding that much-abused term in a genuine mathematical, not a vague biological signification. Here the leader contents himself with a Pisgah prospect, while he sagely thus concludes: "It is wiser not to anticipate (*vorzugreifen*) the future. For the present it is only the *static* of political economy that has been scientifically constructed and has produced useful results."

The leading part which is played by the theory of exchange or law of demand-and-supply in abstract economics is matched by the position of the law of error in the higher statistics. Professor Bortkiewicz shows how that law is applied to the solution of certain problems which we will illustrate by an English example, that which is afforded by Jevons' examination of nearly 100,000 sovereigns circulating in different parts of England. Sorted according to their date, these samples presented the following percentages:—

| | | | | | |
|----------|----------|----------|----------|----------|--------------------|
| 1817-19. | 1820-29. | 1830-39. | 1840-49. | 1850-59. | 1860-67. |
| 0.2 | 7.4 | 7.0 | 16.9 | 28.6 | 38.3. ¹ |

To what degree of accuracy, within what limit of error can we be reasonably sure that if all the sovereigns in England had been counted, the proportions between the numbers of sovereigns bearing dates respectively 1817-19, 1820-29, and so on, would have corresponded to the proportions presented by the sample 100,000? For instance, is it practically certain that the actual number in all England of sovereigns dated 1820-29 was greater than the number of those bearing date 1830-39? How many samples are required to afford this certainty? If the simple treatment of such problems with which Laplace and Poisson were content is to be adopted, it must be granted that the samples were such as would have been obtained if we supposed all the sovereigns in circulation to be collected into one enormous box and a batch of 100,000 coins to be drawn therefrom at random. But things in the concrete are seldom so obligingly simple. Thus, in the case before us, there is reason to believe that the percentages of sovereigns bearing any assigned date were not the same in the different parts of the country where samples were obtained. It is as if the 100,000 samples taken at random were not all taken from one box, but some from one and some from another of several boxes in which the proportions between the number of coins bearing each date were not identical. When there exists this

¹ *Investigations in Currency and Finance*, p. 274. Cp. p. 292. The sum of the percentages above quoted are less than 100 by a matter of 1.6 pertaining to Australian sovereigns.

sort of heterogeneity in the sources from which the samples are derived—this abnormality as it has been called with reference to the simplest species of sortition—then the regulation method of eliminating chance, prescribed too generally by the older mathematicians, becomes, as Professor Bortkiewicz observes, “illusory and worthless.”

In what cases then may this sort of abnormality be expected, on what conditions does it depend? The answer to this interesting question is given in the work before us, read in connection with the author's important treatise *On the Law of Small Numbers*.¹ We may partially illustrate his theory by our example. Supposing that several hundred thousand, instead of one hundred thousand, sample sovereigns had been taken, then the rule proper to the hypothesis of a simple sortition with as it were a single box, becomes less accurate; it is less accurate also when applied to the class of sovereigns dated 1850–9, or that dated 1840–9, than to the much rarer class of sovereigns dated 1817–9, other things being the same. But what those other things are, it is not easy to state with tolerable brevity in plain prose without the aid of symbols.

Considering how frequently the method of eliminating chance prescribed by Laplace proves illusory, we could wish that Professor Bortkiewicz had pronounced more decidedly upon the validity of a substitute which has been proposed. Suppose that a certain class of observed frequencies, such as birth rates for a series of years, do not behave like the proportions in samples taken from a single box, still may we not apply the law of error to this class of statistics for the solution of problems like those above examined, if we employ a coefficient of dispersion—a standard deviation or modulus—determined not on the “combinational” model, to use Professor Lexis' terminology, but according to the “physical” method, the data for which might have been obtained by observing the dispersion of birth rates at different times and places in previous experience?² Doubtless that empirical result would never rest on so large an inductive basis as the combinational coefficient. The procedure too would be at best provisional. We might always hope to break up the material into smaller categories characterised by the more satisfactory species of dispersion.

We have not left ourselves space to consider Professor Bortkiewicz's application of his principles to tables of mortality and sick-

¹ *Das Gesetz der kleiner Zahlen*, Leipzig: Teubner, 1898.

² Cp. *Journal of the Statistical Society*, December 1885.

ness. And we can only allude to the statement of general principles given by another authority, Professor Czuber, in the same volume. His concise treatise on the Calculus of Probabilities (Heft I) forms a good introduction to his well-known larger works.

The History of Trade Unionism. By SIDNEY and BEATRICE WEBB. (London: Longmans, Green & Co. Pp. 558. 8vo.), 1894.

GREAT expectations have naturally been excited by the co-operation of two authors each of whom has, independently of the other, won a high reputation. These expectations are not disappointed, and *The History of Trade Unionism* by Mr. and Mrs. Webb will probably retain in economic literature a place as high as, or even higher than, *The Co-operative Movement* by Miss Beatrice Potter.

This praise will appear deserved, whether we consider the matter or the form of the work.

The gratitude and admiration of all genuine students will be excited by the laborious zeal with which the authors have compiled the materials of their history. Hundreds of references attest their diligence; a bibliography of forty closely-printed pages measures the extent of their researches. Obscure pamphlets and defunct newspapers have been scrutinised, remote provincial libraries have been put under contribution. It is tantalising to hear of many authorities which are not to be found in the British Museum; for instance, E. C. Tuffnell's *Character and Effects of Trades Unions* (1834), which is described as "perhaps the best statement of the case against Trade Unionism."

The writers obtain a great advantage from having cultivated the acquaintance of the living leaders of the movement which forms their subject. From this source has been derived much information inaccessible to the general public. In many a working man's home, we are told, the descendants of the old skilled handicraftsmen have unearthed "grandfather's indentures," or "father's old card," or a tattered "set of rules." In short, so great efforts, and with so much success, have seldom been made to dissipate the obscurity which envelops *les origines*.

The mass of material would have proved unmanageable in less skilful hands. A crowd of homogeneous events, the homely incidents of industrial discord—for the most part unrelieved by the interest which attaches to commanding personalities—would not have impressed our imagination, nor have been retained in